THE ORNL MATHEMATICAL PHANTOM SERIES

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Dec., 1996

INTRODUCTION

The ORNL mathematical phantom series was designed by Cristy (1980) after the adult phantom of Snyder *et al.* (1978). Further developments were detailed by Cristy and Eckerman (1987) and by Eckerman and Ryman (1993). This document assembles in one place the mathematical description of the series as of the date shown above. For information on the methods used to develop the phantom series and references to anatomical data the above reports should be consulted.

MATHEMATICAL DESCRIPTION OF THE PHANTOMS

The phantom series includes the newborn and individuals of ages 1, 5, 10, 15, and the adult. The age 15 phantom represents both a 15-year-old male and an adult female. The adult phantom is labeled here as 'adult male,' although it, as all members, is hermaphroditic. The exterior of each phantom has approximately the form of the human body, but there has been no attempt to introduce small variations which would be presumed to have only a small effect on the scattering of photons. Similarly, the descriptions of the interior organs, while approximately correct as to size, shape, position, composition and density, are simplified to provide formulas which are readily evaluated by a digital computer.

Each phantom consists of three major sections: (1) an elliptical cylinder representing the trunk and arms; (2) two truncated circular cones representing the legs and feet; and (3) a circular cylinder on which sits an elliptical cylinder capped by half an ellipsoid representing the neck and head. Attached to the male legs section is a small region with a planar front surface to contain the testes. Attached to the trunk are portions of two ellipsoids representing the female breasts.

Elemental Composition of Tissues

In the phantoms three tissue types are recognized: skeletal, lung, and all other tissue. The elemental composition of each tissue type is given in Table 1 along with the densities. It is generally acknowledged that the elemental composition and specific gravity of the newborn are different from those of the adult. A higher water content and lower bone mineral content are the most prominent differences.

Description of the Body Regions and Organs

Body regions

The body is represented as erect with the positive z-axis directed upward toward the head. The x-axis is directed to the phantom's left, and the y-axis is directed toward the posterior side of the phantom. The origin is taken at the center of the base of the trunk section of the phantom. Dimensions (in centimeters) are stated here to two decimal places. The use of two decimal places does not imply that the average dimensions in some reference population are known to such precision. This use is for convenience in designing the organs with correct volumes and spatial relationships.

Trunk. The trunk, exclusive of the female breasts, is represented by a solid elliptical cylinder specified by

$$\left(\frac{x}{A_T}\right)^2 + \left(\frac{y}{B_T}\right)^2 \le 1 \text{ and } 0 \le z \le C_T.$$

The values of A_T, B_T, and C_T for each phantom are given in the table below.

Phantom	Length (cm) A_T	B_T	C_T	Volume (cm^3)	Mass (g)
Newborn	6.35	4.90	21.60	2,050	2,030
Age 1	8.80	6.50	30.70	5,350	5,350
Age 5	11.45	7.50	40.80	10,660	10,650
Age 10	13.90	8.40	50.80	18,050	18,130
15-AF	17.25	9.80	63.10	32,920	33,500
Adult male	20.00	10.00	70.00	43,090	43,470

The trunk section includes the arms and the pelvic region to the crotch. The female breasts are appended to the outside of the trunk section. The volumes and masses for the trunk given above do not include the breasts.

Head. The head section includes a neck, represented by a right circular cylinder, and the head, consisting of a right elliptical cylinder topped by half an ellipsoid. The neck is specified by

$$x^2 + y^2 \le R_H^2 \quad \text{and} \quad C_T \le z \le C_T + C_{H0}$$

The head is specified by

$$\left(\frac{x}{A_H}\right)^2 + \left(\frac{y}{B_H}\right)^2 \le 1 \text{ and } C_T + C_{H0} \le z \le C_T + C_{H0} + C_{H1},$$

or

$$\left(\frac{x}{A_H}\right)^2 + \left(\frac{y}{B_H}\right)^2 + \left(\frac{z - (C_T + C_{H0} + C_{H1})}{C_{H2}}\right)^2 \le 1 \text{ and } z > C_T + C_{H0} + C_{H1}.$$

Phantom	Length (cm) R_H	A_H	B_H	C_{HO}	C_{H1}	C_{H2}	Volume (cm^3)	Mass (g)
Newborn	2.8	4.52	5.78	1.50	7.01	3.99	831	876
Age 1	3.6	6.13	7.84	2.30	9.50	5.41	2,070	2,220
Age 5	3.8	7.13	9.05	3.30	10.70	6.31	3,170	3,480
Age 10	4.4	7.43	9.40	4.70	11.68	6.59	3,810	4,210
15-AF	5.2	7.77	9.76	7.70	12.35	6.92	4,700	5,220
Adult male	5.4	8.00	10.00	8.40	13.05	7.15	5,250	5,870

The values of C_T have been given previously in the table of trunk values.

Legs. The legs region of each phantom consists of the frustrums of two circular cones specified by

$$x^2 + y^2 \le \pm x\left(A_T + \frac{A_T}{C_L} z\right) \quad \text{and} \quad -C_L \le z \le 0,$$

where the "±" sign is taken as plus for the left leg and minus for the right leg.

Phantom	Length (cm) C_L	C'_L	Volume (cm^3)	Mass (g)
Newborn	16.8	21.6	451	480
Age 1	26.5	37.1	1,470	1,600
Age 5	48.0	65.0	4,380	4,780
Age 10	66.0	90.0	8,930	9,740
15-AF	78.0	100.0	15,400	16,800
Adult male	80.0	100.0	20,800	22,600

The values of A_T have been given previously in the table of trunk values.

Male genitalia. The male genitalia region of each phantom consists of the region specified by

$$z_1 \le z \le 0, \; -r \le x \le r, \; -r \le y \le 0, \text{ and}$$

$$(x \pm r)^2 + y^2 \ge r^2.$$

The last inequality must hold for either choice of sign (i.e., the genitalia region lies outside both legs). The

value of r is given by the expression $0.5A_T(1 + z/C'_L)$, where A_T is the trunk dimension and C'_L is the legs dimension defined previously. The value of z_1 is given by the expression $-(2c + S)$, where c is the value defined for the testes and S is the skin thickness. Thus, all of the parametric values are defined elsewhere, and only the volumes are given here.

Phantom	Volume (cm^3)
Newborn	5.48
Age 1	12.1
Age 5	23.2
Age 10	36.2
15-AF	109
Adult male	196

Organs

In the equations of the organs, which follow, the body section parameters A_T, B_T, C_T, A_H, B_H, C_{H1}, C_{H2}, C_L, and C'_L and the skin thickness S will be used without further explanation or denotation. Symbols for other parameters, usually lowercase letters, have meaning only for the organ being defined. The symbol "a," for example, is used in defining many different organs. The volume of the organ will be given with the parameter values. The mass determined by this volume and the appropriate density is given in Appendix A.

Skeletal System. The skeletal system consists of the 13 parts described below.

Leg bones. Each leg bone is the frustrum of a circular cone. In the defining inequalities below, the "±" sign is taken as minus for the left leg bone and plus for the right:

$$\left[x \pm \left(\frac{A_T}{2} + \frac{k\,z}{C_L - S}\right)\right]^2 + y^2 \le \left[R_1 + \left(\frac{R_1 - R_2}{C_L - S}\right) z\right]^2,$$

$$\text{and} \quad -(C_L - S) \le z \le 0,$$

in which

$$k = \frac{A_T}{2}\left(1 - \frac{C_L' - C_L}{C_L'}\right),\ R_1 = 0.175\,A_T,\ \text{and}\ R_2 = \frac{A_T}{4}\left(\frac{C_L' - C_L}{C_L'}\right).$$

Phantom	Volume (both) (cm^3)
Newborn	61.4
Age 1	207
Age 5	610
Age 10	1250
15 - AF	2100
Adult male	2800

Arm bones. Each arm bone is the frustrum of an elliptical cone and is defined by

$$\left[\frac{\left(\frac{a}{2z_2}\right)(z - z_2) + (x - x_0)}{a}\right]^2 + \left(\frac{y}{b}\right)^2 \le \left[\frac{2z_2 + (z - z_2)}{2z_2}\right]^2$$

$$and\ 0 \le z \le z_2.$$

In the table below, positive values of x_0 are used for the left arm bone and negative for the right. The volume is that of both arm bones.

Phantom	a	b	x_o	z_2	Volume (cm^3)
Newborn	0.44	1.32	±5.84	21.29	45.3
Age 1	0.62	1.76	±8.10	30.26	121
Age 5	0.80	2.03	±10.53	40.22	239
Age 10	0.97	2.27	±12.79	50.07	404
15-AF	1.21	2.65	±15.87	62.20	731
Adult male	1.40	2.70	±18.40	69.00	956

Pelvis. The pelvis is a portion of the volume between two nonconcentric elliptical cylinders. The inequalities defining the pelvis are

$$\left(\frac{x}{a_2}\right)^2 + \left(\frac{y - y_{02}}{b_2}\right)^2 \le 1\ ,$$

$$\left(\frac{x}{a_1}\right)^2 + \left(\frac{y - y_{01}}{b_1}\right)^2 \ge 1\ ,$$

$$y \ge y_{02},\ 0 \le z \le z_2,\ and\ y \le y_1\ if\ z \le z_1.$$

Phantom	a_1	b_1	a_2	b_2	y_{01}	y_{02}	y_1	z_1	z_2	Volume (cm^3)
Newborn	3.59	5.54	3.81	5.88	-1.86	-1.47	2.45	4.32	6.79	28.9
Age 1	4.97	7.35	5.28	7.80	-2.47	-1.95	3.25	6.14	9.65	76.0
Age 5	6.47	8.48	6.87	9.00	-2.85	-2.25	3.75	8.16	12.82	151
Age 10	7.85	9.49	8.34	10.08	-3.19	-2.52	4.20	10.16	15.97	258
15-AF	9.75	11.07	10.35	11.76	-3.72	-2.94	4.90	12.62	19.83	460
Adult male	11.30	11.30	12.00	12.00	-3.80	-3.00	5.00	14.00	22.00	606

Spine. The spine is an elliptical cylinder given by

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y - y_0}{b}\right)^2 \le 1 \quad and \quad z_1 \le z \le z_4.$$

It is divided into 3 portions – an upper, middle, and lower – such that dose and absorbed fractions can be estimated separately for each portion. These divisions are formed by the planes $z = z_2$ and $z = z_3$. The upper portion of the spine, within the neck, is displaced forward from the location within the trunk. The parameter $y_0^{/}$ replaces y_0 for $z_3 \le z \le z_4$.

Phantom	a	b	y_0	$y_0^{'}$	z_1	z_2	z_3	z_4	Volume (cm^3)
Newborn	0.64	1.23	2.70	0.40	6.79	10.83	21.53	26.30	48.1
Age 1	0.88	1.63	3.58	0.20	9.65	15.39	30.62	37.29	124
Age 5	1.15	1.88	4.13	0.69	12.82	20.46	40.71	48.72	243
Age 10	1.39	2.10	4.62	0.90	15.97	25.47	50.70	60.84	411
15-AF	1.73	2.45	5.39	1.00	19.83	31.64	62.93	76.66	754
Adult male	2.00	2.50	5.50	1.45	22.00	35.10	69.80	84.80	983

Skull. The skull comprises the cranium and the facial skeleton. The cranium is represented by the volume between two concentric ellipsoids defined by

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z - (C_T + C_{H0} + C_{H1})}{c}\right)^2 \ge 1$$

$$and \left(\frac{x}{a + d}\right)^2 + \left(\frac{y}{b + d}\right)^2 + \left(\frac{z - (C_T + C_{H0} + C_{H1})}{c + d}\right)^2 \le 1.$$

The values a, b, and c are the same as the values a, b, and c given in the statements and table for the brain.

Phantom	d	Volume (cm^3)
Newborn	0.20	49.8
Age 1	0.30	139
Age 5	0.56	339
Age 10	0.67	434
15-AF	0.76	508
Adult male	0.90	618

The facial skeleton is represented by a portion of the volume between two concentric elliptical cylinders. The portion of the volume that intersects the cranium and brain is excluded. The inequalities are

$$\left(\frac{x}{a_1}\right)^2 + \left(\frac{y}{b_1}\right)^2 \le 1,$$

$$\left(\frac{x}{a_1 - d}\right)^2 + \left(\frac{y}{b_1 - d}\right)^2 \ge 1,$$

$$y \le 0,\ C_T + C_{H0} + z_1 \le z \le C_T + C_{H0} + z_5,$$

$$and\ \left(\frac{x}{a_2}\right)^2 + \left(\frac{y}{b_2}\right)^2 + \left(\frac{z - (C_T + C_{H0} + C_{H1})}{c_2}\right)^2 > 1.$$

The variables a_2, b_2, and c_2 correspond in numerical values with the variable expressions *(a + b)*, *(b + d)*, and *(c + d)*, respectively, in the statements defining the cranium and hence are not given below.

Phantom	a_1	b_1	d	z_1	z_5	Volume (cm^3)
Newborn	4.17	5.43	0.07	2.16	8.18	6.13
Age 1	5.73	7.44	0.14	2.93	11.18	22.8
Age 5	6.68	8.60	0.58	3.30	12.57	114
Age 10	6.93	8.90	0.74	3.61	13.73	161
15-AF	6.92	8.91	1.10	3.79	14.05	234
Adult male	7.00	9.00	1.40	4.00	14.73	305

Rib cage. The rib volume is a series of bands between two concentric, right-vertical, elliptical cylinders. This region is sliced by a series of equispaced horizontal planes into slabs, every other slice being a rib. The statements that must be satisfied are

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 \le 1,$$

$$\left(\frac{x}{a-d}\right)^2 + \left(\frac{y}{b-d}\right)^2 \ge 1,$$

$$z_1 \le z \le z_2 \text{ and } Int\left(\frac{z-z_1}{c}\right) \text{ is even.}$$

The function *Int* (*u*) is the integral part of *u* [e.g., *Int* (3.67) = 3]. Thus, the statement "*Int* [(z - z_1])/c] is even" amounts to requiring that

$$0 \le \frac{z-z_1}{c} < 1 \quad or \quad 2 \le \frac{z-z_1}{c} \le 3 \quad or \quad 4 \le \frac{z-z_1}{c} < 5, \text{ etc.}$$

Phantom	*a*	*b*	*d*	z_1	z_2	*c*	Volume (cm^3)
Newborn	5.40	4.80	0.21	10.86	20.75	0.43	34.0
Age 1	7.48	6.37	0.28	15.44	29.47	0.61	87.4
Age 5	9.73	7.35	0.34	20.53	39.16	0.81	174
Age 10	11.82	8.23	0.39	25.43	48.89	1.02	295
15-AF	14.66	9.60	0.47	31.67	60.65	1.26	531
Adult male	17.00	9.80	0.50	35.10	67.30	1.40	694

Clavicles. The clavicles are represented as two portions of a torus which lie along the circular arc $x^2 + (y - y_0)^2 = R^2$ at $z = z_1$ and have a smaller radius of *r*. The clavicles include only the portion of the torus between the planes $y_0 - y = |x| \cot \theta_1$ and $y_0 - y = |x| \cot \theta_2$. (The absolute value sign on *x* allows for both a right and a left clavicle.) These equations can be reduced to the form

$$(z - z_1)^2 + \left(R - \sqrt{x^2 + (y - y_0)^2}\right)^2 \le r^2 ,$$

$$\cot \theta_2 \le \frac{y_0 - y}{|x|} \le \cot \theta_1, \quad and \quad y < 0 \quad .$$

Phantom	y_0	z_1	*R*	*r*	cot θ_1	cot θ_2	Volume (cm^3)
Newborn	0.73	21.06	5.07	0.2833	5.5868	0.38510	2.62
Age 1	1.38	29.93	7.14	0.3930	5.6814	0.43161	6.85
Age 5	3.14	39.78	9.80	0.4491	5.9977	0.56391	13.7
Age 10	4.93	49.53	12.40	0.5981	6.2581	0.65708	23.2
15-AF	7.22	61.52	15.93	0.7274	6.4852	0.73137	41.6
Adult male	11.10	68.25	20.00	0.7883	7.0342	0.89415	54.7

The volume tabulated above is for both clavicles. The clavicles lie slightly inside the cylinder defining the rib cage and just above the top rib.

Scapulae. The scapulae are defined as part of the volume between two concentric elliptical cylinders. For each scapula, the volume is bounded by the planes $z = z_1$, $z = z_2$, $y = m_1/x|$, and $y = m_2/x|$. (The absolute value sign on x allows for both a right and a left scapula.) The defining inequalities are

$$\left(\frac{x}{a_2}\right)^2 + \left(\frac{y}{b}\right)^2 \le 1,$$

$$\left(\frac{x}{a_1}\right)^2 + \left(\frac{y}{b}\right)^2 > 1,$$

$$z_1 \le z \le z_2,\ y > 0,\ and\ m_1 < \frac{y}{|x|} < m_2.$$

Phantom	a_1	a_2	b	z_1	z_2	m_1	m_2	Volume (cm^3)
Newborn	5.40	6.04	4.80	15.71	20.77	0.39	1.23	9.64
Age 1	7.48	8.36	6.37	22.32	29.52	0.37	1.18	25.3
Age 5	9.73	10.88	7.35	29.67	39.23	0.33	1.05	50.4
Age 10	11.82	13.20	8.23	36.94	48.84	0.30	0.97	85.7
15-AF	14.66	16.36	9.60	45.88	60.67	0.28	0.91	154
Adult male	17.00	19.00	9.80	50.90	67.30	0.25	0.80	202

Bone marrow. The regional distributions of the active (hematopoietic) bone marrow and the inactive (fatty) marrow vary greatly with age. The approximate weights of the total (active plus inactive) marrow, the active marrow, and the inactive marrow as a function of age are:

	Marrow Mass (g)		
Phantom	Total	Active	Inactive
Newborn	47	47	0
Age 1	170	150	20
Age 5	460	320	140
Age 10	1200	610	590
15-AF	2600	1050	1550
Adult male	3500	1120	2380

The active marrow in bone groups of the phantoms, expressed as the percentage of active marrow in the body, are given in Table 2. Similarly, in Table 3, are given the inactive (fatty) marrow percentages. The marrow, active or inactive, is assumed to be distributed uniformly in the bone regions defined. The total mass of the skeleton in each phantom is given in the following table.

Phantom	Mass of skeleton (g)
Newborn	351
Age 1	1,140
Age 5	2,710
Age 10	4,630
15-AF	7,650
Adult male	10,000

Adrenals. Each adrenal is half an ellipsoid atop a kidney, defined by

$$\left(\frac{x_1}{a}\right)^2 + \left(\frac{y_1}{b}\right)^2 + \left(\frac{z_1}{c}\right)^2 \le 1 \quad \textit{and} \quad z_1 \ge 0,$$

where the (x_1, y_1, z_1)-coordinate system is related to the phantom's (x, y, z)-coordinate system by the following rotation-translation equations, given in matrix form:

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x - x_0 \\ y - y_0 \\ z - z_0 \end{bmatrix}$$

In the following table of parametric values, x_0 and θ are both taken as positive for the left adrenal, and both negative for the right. The volumes tabulated below are for both adrenals.

Phantom	a	b	c	x_o	y_o	z_o	θ	Volume (cm^3)
Newborn	1.61	0.54	1.54	±1.41	2.45	11.73	±63.3	5.61
Age 1	1.05	0.35	2.20	±1.54	3.25	16.66	±62.2	3.39
Age 5	1.12	0.37	2.92	±2.00	3.75	22.14	±59.3	5.07
Age 10	1.17	0.39	3.63	±2.43	4.20	27.58	±57.2	6.94
15-AF	1.30	0.43	4.30	±3.02	4.90	34.26	±55.6	10.1
Adult male	1.50	0.50	5.00	±3.50	5.00	38.00	±52.0	15.7

Brain. The brain is an ellipsoid given by

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z - (C_T + C_{H0} + C_{HI})}{c}\right)^2 \le 1.$$

Phantom	*a*	*b*	*c*	Volume (cm^3)
Newborn	4.14	5.40	3.61	338
Age 1	5.63	7.34	4.91	850
Age 5	6.34	8.26	5.52	1210
Age 10	6.51	8.48	5.67	1310
15-AF	6.58	8.57	5.73	1350
Adult male	6.60	8.60	5.75	1370

Breasts. The female breasts are represented by portions of two ellipsoids attached to the trunk, given by

$$\left(\frac{x - x_0}{a}\right)^2 + \left(\frac{y - y_0}{b}\right)^2 + \left(\frac{z - z_0}{c}\right)^2 \le 1$$

$$\text{and} \quad \left(\frac{x}{A_T}\right)^2 + \left(\frac{y}{B_T}\right)^2 > 1, \text{ where } y_0 = -B_T\sqrt{1 - \left(\frac{x_0}{A_T}\right)^2}\ .$$

The positive values of x_0 in the table below are taken for the left breast; and the negative values, for the right breast. Since the outer thickness S is counted as skin, the breast tissue is represented by

$$\left(\frac{x - x_0}{a - S}\right)^2 + \left(\frac{y - y_0}{b - S}\right)^2 + \left(\frac{z - z_0}{c - S}\right)^2 \le 1$$

$$\text{and} \left(\frac{x}{A_T}\right)^2 + \left(\frac{y}{B_T}\right)^2 > 1.$$

In the tabulation below, the recall that the 'Adult male' phantom is hermaphroditic. Thus, the volume stated is that of female breast tissue.

						Volume (both) (cm^3)	
Phantom	*a*	*b*	*c*	x_o	z_o	Including skin	Excluding skin
Newborn	0.36	0.36	0.36	3.18	16.05	0.197	0.103
Age 1	0.63	0.63	0.63	4.40	22.81	1.06	0.704
Age 5	0.79	0.79	0.79	5.73	30.31	2.09	1.45
Age 10	0.94	0.94	0.94	6.95	37.73	3.51	2.50
15-AF	4.95	4.35	4.15	8.63	46.87	391	347
Adult male	4.95	4.35	4.15	10.00	52.00	388	337

Gall bladder and contents. For the age 1, the age 5, the age 10, the age-15-male/adult-female, and the adult male phantoms, the gall bladder is represented by the frustrum of a cone capped with a hemisphere. For the newborn phantom, the gall bladder is cylindrical. The gall bladder is defined as a walled organ.

The equations are given below in (x_1, y_1, z_1)-coordinates which are related to (x, y, z)-coordinate system by the following rotation-translation equations:

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} \alpha_1 & \beta_1 & \gamma_1 \\ \alpha_2 & \beta_2 & \gamma_2 \\ \alpha_3 & \beta_3 & \gamma_3 \end{bmatrix} \begin{bmatrix} x - x_0 \\ y - y_0 \\ z - z_0 \end{bmatrix}.$$

The walls are specified as follows:
(hemispherical part)

$$x_1^2 + y_1^2 + z_1^2 \le r_2^2 ,$$
$$x_1^2 + y_1^2 + z_1^2 \ge r_1^2 ,$$
$$\text{and} \quad z_1 < 0;$$

and the (conical part)

$$x_1^2 + y_1^2 \le (r_2 - sz_1)^2$$
$$x_1^2 + y_1^2 \ge (r_1 - sz_1)^2$$
$$\text{and } 0 \le z_1 \le h$$

The contents are specified as follows:
(hemispherical part)

$$x_1^2 + y_1^2 + z_1^2 < r_1^2 \text{ and } \quad z_1 < 0;$$

and (conical part)

$$x_1^2 + y_1^2 < (r_1 - sz_1)^2 \text{ and } 0 \le z_1 \le h.$$

To obtain the equations for the newborn gall bladder wall and contents, set s = 0 and ignore the hemispherical part.

Phantom	α_1	β_1	γ_1	α_2	β_2	γ_2	α_3	β_3	γ_3
Newborn	0.9292	0	-0.3695	-0.1018	0.9613	-0.2559	0.3553	0.2754	0.8933
Age 1	0.9770	0	-0.2132	-0.0348	0.9866	-0.1594	0.2105	0.1632	0.9639
Age 5	0.9814	0	-0.1921	-0.0291	0.9884	-0.1490	0.1898	0.1518	0.9700
Age 10	0.9722	0	-0.2342	-0.0400	0.9853	-0.1661	0.2307	0.1709	0.9579
15-AF	0.9550	0	-0.2964	-0.0606	0.9789	-0.1952	0.2903	0.2044	0.9349
Adult male	0.9615	0	-0.2748	-0.0574	0.9779	-0.2008	0.2687	0.2090	0.9403

Phantom	r_1	r_2	s	h	x_0	y_0	z_0
Newborn	0.458	0.500	0	2.80	-0.67	-1.75	8.68
Age 1	0.884	0.937	0.2275	3.54	-0.71	-2.08	13.16
Age 5	1.414	1.499	0.2275	5.66	-0.59	-2.40	17.49
Age 10	1.768	1.874	0.2275	7.07	-1.69	-2.69	21.77
15-AF	1.916	2.031	0.2275	7.66	-3.98	-3.14	27.04
Adult male	2.000	2.120	0.2275	8.00	-4.50	-3.20	30.00

	Volume (cm^3)		
Phantom	Wall	Contents	Total
Newborn	0.354	1.85	2.20
Age 1	0.875	4.62	5.50
Age 5	3.59	18.9	22.5
Age 10	7.00	37.0	44.0
15-AF	8.92	47.1	56.0
Adult male	10.1	53.6	63.7

Gastrointestinal tract and contents.

Esophagus. The esophagus model is divided into two parts. The thoracic portion lies within the upper trunk and contains a small amount of air (represented by a void in the model). It is described as the region between two right elliptical cylinders:

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y - y_0}{b}\right)^2 \le 1$$

$$\left(\frac{x}{a - d}\right)^2 + \left(\frac{y - y_0}{b - d}\right)^2 \ge 1$$

where $z_2 \le z \le z_3$. The abdominal portion lies within the upper abdomen (middle trunk of the phantom) and is closed. It is described as a right circular cylinder:

$$y'^2 + z'^2 \le r^2 , \quad x_1' \le x' \le x_2'$$

where a point in the (x', y', z') coordinate system is related to the phantoms's (x, y, z) coordinate system by the following rotation-translation equation:

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \alpha_1 & \beta_1 & \gamma_1 \\ \alpha_2 & \beta_2 & \gamma_2 \\ \alpha_3 & \beta_3 & \gamma_3 \end{bmatrix} \begin{bmatrix} x \\ y - y_0 \\ z - z_1 \end{bmatrix}$$

Age	a	b	d	y_0	z_2	z_3	r	x_1'	x_2'	z_1
Newborn	0.35	0.21	0.14	1.15	13.27	21.6	0.23	0.00	3.03	12.93
1 yr old	0.51	0.27	0.19	1.33	18.86	30.7	0.34	0.00	3.86	18.40
5 yr old	0.65	0.32	0.22	1.69	25.06	40.8	0.44	0.00	4.84	24.53
10 yr old	0.79	0.36	0.25	2.04	31.21	50.8	0.52	0.00	5.75	30.62
15 yr old	1.05	0.40	0.28	2.29	38.76	63.1	0.64	0.00	7.07	38.08
Adult	1.17	0.42	0.30	2.575	43.0	70.0	0.70	0.10	7.80	42.30

Age	α_1	β_1	γ_1	α_2	β_2	γ_2	α_3	β_3	γ_3
Newborn	0.571348	-0.791789	-0.215943	0.810922	0.585154	0.0	0.126360	-0.175113	0.976406
1 yr old	0.639520	-0.732178	-0.234370	0.753155	0.657843	0.0	0.154178	-0.176517	0.972148
5 yr old	0.663545	-0.701214	-0.260782	0.726347	0.687328	0.0	0.179243	-0.189418	0.965398
10 yr old	0.678998	-0.677776	-0.282102	0.706470	0.707743	0.0	0.199655	-1.199296	0.959385
15 yr old	0.708385	-0.637547	-0.302860	0.668965	0.743294	0.0	0.225114	-0.202603	0.953035
Adult	0.736084	-0.604969	-0.303634	0.634945	0.772557	0.0	0.234575	-0.192791	0.952789

Phantom	Volume (cm^3)
Newborn	2.04
Age 1	5.57
Age 5	11.1
Age 10	18.7
15-AF	34.1
Adult male	44.7

Stomach. The stomach wall is represented by the volume between two concentric ellipsoids. The contents are represented by the volume within the inner ellipsoid. The wall is defined by

$$\left(\frac{x - x_0}{a}\right)^2 + \left(\frac{y - y_0}{b}\right)^2 + \left(\frac{z - z_0}{c}\right)^2 \leq 1$$

$$\text{and } \left(\frac{x - x_0}{a - d}\right)^2 + \left(\frac{y - y_0}{b - d}\right)^2 + \left(\frac{z - z_0}{c - d}\right)^2 \geq 1.$$

The contents are defined by

$$\left(\frac{x - x_0}{a - d}\right)^2 + \left(\frac{y - y_0}{b - d}\right)^2 + \left(\frac{z - z_0}{c - d}\right)^2 < 1 .$$

Phantom	a	b	c	d	x_0	y_0	z_0
Newborn	1.20	1.39	2.34	0.22	2.54	-1.96	10.80
Age 1	1.85	2.05	3.51	0.33	3.52	-2.70	15.35
Age 5	2.55	2.40	4.66	0.45	4.58	-3.15	20.40
Age 10	3.14	2.74	5.81	0.53	5.56	-3.51	25.40
15-AF	3.43	2.92	7.16	0.56	6.90	-3.92	31.55
Adult male	4.00	3.00	8.00	0.613	8.00	-4.00	35.00

	Volume (cm^3)	
Phantom	Wall	Contents
Newborn	6.17	10.2
Age 1	20.9	34.8
Age 5	47.2	72.2
Age 10	81.8	128
15-AF	113	187
Adult male	152	250

The stomach represented here is a "full" stomach, and the average dose rate, even for the same activity present, probably varies greatly depending on the degree of extension of the stomach, presence of air spaces, etc.

Small intestine. The small intestine does not seem to remain in any "standard position" except the ends, which are relatively fixed. Thus, the small intestine is to be regarded as occupying a volume within which it is free to move. No attempt to determine a specific configuration is made here, and thus the wall and contents are not distinguished for estimation of photon dose. The small intestine and contents are represented by a section of an elliptical cylinder, defined by

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y - y_0}{b}\right)^2 \leq 1 ,$$

$$y_1 \leq y \leq y_2 \text{ and } z_1 \leq z \leq z_2.$$

The portion of the large intestine within this region is excluded.

Phantom	a	b	y_0	y_1	y_2	z_1	z_2	Volume (cm^3)
Newborn	3.59	5.54	-1.86	-2.39	1.08	5.25	8.33	50.9
Age 1	4.97	7.35	-2.47	-3.16	1.43	7.46	11.84	132
Age 5	6.47	8.48	-2.85	-3.65	1.65	9.91	15.74	265
Age 10	7.85	9.49	-3.19	-4.08	1.85	12.34	19.59	447
15-AF	9.75	11.07	-3.72	-4.76	2.16	15.32	24.34	806
Adult male	11.30	11.30	-3.80	-4.86	2.20	17.00	27.00	1060

Upper large intestine. The upper large intestine consists of an ascending colon and a transverse colon. The ascending colon wall is defined by the space between two coaxial elliptical cylinders:

$$\left(\frac{x - x_0}{a}\right)^2 + \left(\frac{y - y_0}{b}\right)^2 \le 1,$$

$$\left(\frac{x - x_0}{a - d}\right)^2 + \left(\frac{y - y_0}{b - d}\right)^2 \ge 1,$$

$$\text{and} \quad z_1 \le z \le z_2.$$

The contents are defined by the space within the inner cylinder,

$$\left(\frac{x - x_0}{a - d}\right)^2 + \left(\frac{y - y_0}{b - d}\right)^2 < 1$$

$$\text{and } z_1 \le z \le z_2.$$

								Volume (cm^3)	
Phantom	a	b	d	x_0	y_0	z_1	z_2	Wall	Contents
Newborn	0.79	1.23	0.27	-2.70	-1.16	4.46	7.41	4.38	4.63
Age 1	1.10	1.63	0.37	-3.74	-1.53	6.34	10.53	11.5	12.1
Age 5	1.43	1.88	0.46	-4.87	-1.77	8.42	13.99	22.9	24.1
Age 10	1.74	2.10	0.54	-5.91	-1.98	10.49	17.42	38.8	40.8
15-AF	2.16	2.45	0.65	-7.33	-2.31	13.03	21.63	69.5	73.4
Adult male	2.50	2.50	0.7085	-8.50	-2.36	14.45	24.00	91.2	96.3

The transverse colon wall is also defined by the space between two coaxial elliptical cylinders:

$$\left(\frac{y - y_0}{b}\right)^2 + \left(\frac{z - z_0}{c}\right)^2 \leq 1,$$

$$\left(\frac{y - y_0}{b - d}\right)^2 + \left(\frac{z - z_0}{c - d}\right)^2 \geq 1,$$

$$and \quad -x_1 \leq x \geq x_1.$$

The contents are defined by the space within the inner cylinder,

$$\left(\frac{y - y_0}{b - d}\right)^2 + \left(\frac{z - z_0}{c - d}\right)^2 < 1$$

$$and \quad -x_1 \leq x \leq x_1.$$

							Volume (cm^3)	
Phantom	b	c	d	y_0	z_0	x_1	Wall	Contents
Newborn	1.23	0.46	0.18	-1.16	7.87	3.33	5.69	6.15
Age 1	1.63	0.65	0.26	-1.53	11.18	4.62	15.2	15.5
Age 5	1.88	0.87	0.33	-1.77	14.86	6.01	30.2	31.6
Age 10	2.10	1.08	0.40	-1.98	18.51	7.30	51.0	53.0
15-AF	2.45	1.35	0.49	-2.31	22.99	9.06	92.3	96.0
Adult male	2.50	1.50	0.527	-2.36	25.50	10.50	121	127

Lower large intestine. The lower large intestine consists of a descending colon and a sigmoid colon. The descending colon wall is defined by the space between two coaxial elliptical cylinders. The axis of the cylinders is at a slight angle with the z-axis of the phantom, but the ends of the descending colon are defined by horizontal planes *($z = z_1$* and *$z = z_2$)*. The wall is specified by

$$\left(\frac{x - x_0}{a}\right)^2 + \left(\frac{y - y_0}{b}\right)^2 \leq 1,$$

$$\left(\frac{x - x_0}{a - d}\right)^2 + \left(\frac{y - y_0}{b - d}\right)^2 \geq 1,$$

$$and \quad z_1 \leq z \leq z_2,$$

where

$$x_0 = x_1 + \frac{m_x (z - z_2)}{z_2 - z_1} \quad and \quad y_0 = \frac{m_y (z_1 - z)}{z_2 - z_1}$$

The contents of the descending colon are defined by the space within the inner cylinder, i.e.,

$$\left(\frac{x - x_0}{a - d}\right)^2 + \left(\frac{y - y_0}{b - d}\right)^2 < 1,$$

$$and \; z_1 \le z \le z_2.$$

									Volume (cm^3)	
Phantom	a	b	d	x_1	m_x	m_y	z_1	z_2	Wall	Contents
Newborn	0.60	1.04	0.20	2.94	0.2477	1.225	2.69	7.41	4.27	4.98
Age 1	0.83	1.38	0.27	4.07	0.3432	1.625	3.82	10.53	11.0	13.1
Age 5	1.08	1.60	0.34	5.30	0.4466	1.875	5.08	13.99	22.3	26.1
Age 10	1.31	1.79	0.40	6.43	0.5421	2.100	6.33	17.42	37.6	44.1
15-AF	1.62	2.09	0.49	7.98	0.6728	2.450	7.86	21.63	68.3	78.2
Adult male	1.88	2.13	0.54	9.25	0.7800	2.500	8.72	24.00	89.9	102

The sigmoid colon plus contents is represented by portions of two flattened tori; that is, the axis of each torus is circular but the cross-section is elliptical. The wall is defined as follows:

(portion of upper torus)

$$\left(\frac{\sqrt{(x - x_0)^2 + (z - z_0)^2} - R_1}{a}\right)^2 + \left(\frac{y}{b}\right)^2 \le 1 \;,$$

$$\left(\frac{\sqrt{(x - x_0)^2 + (z - z_0)^2} - R_1}{a - d}\right)^2 + \left(\frac{y}{b - d}\right)^2 \ge 1,$$

$$x \ge x_0, \quad and \quad z \le z_0;$$

and (portion of lower torus)

$$\left(\frac{\sqrt{(x - x_0)^2 + z^2} - R_2}{a}\right)^2 + \left(\frac{y}{b}\right)^2 \le 1,$$

$$\left(\frac{\sqrt{(x - x_0)^2 + z^2} - R_2}{a - d}\right)^2 + \left(\frac{y}{b - d}\right)^2 \ge 1,$$

$$x \le x_0, \quad and \quad z \ge 0.$$

The contents of the sigmoid colon are defined as follows:

(portion of upper torus)

$$\left(\frac{\sqrt{(x - x_0)^2 + (z - z_0)^2} - R_1}{a - d} \right)^2 + \left(\frac{y}{b - d} \right)^2 < 1,$$

$$x \geq x_0 \quad and \quad z \leq z_0;$$

and (portion of lower torus)

$$\left(\frac{\sqrt{(x - x_0)^2 + z^2} - R_2}{a - d} \right)^2 + \left(\frac{y}{b - d} \right)^2 < 1,$$

$$x \leq x_0, \quad and \quad z \geq 0.$$

								Volume (cm^3)	
Phantom	a	b	d	x_0	z_0	R_1	R_2	Wall	Content
Newborn	0.50	0.77	0.25	0.95	2.69	1.77	0.92	3.39	1.73
Age 1	0.69	1.02	0.34	1.32	3.82	2.51	1.31	8.78	4.49
Age 5	0.88	1.21	0.42	1.72	5.08	3.33	1.75	17.6	9.11
Age 10	0.96	1.50	0.48	2.09	6.33	4.15	2.18	29.7	15.3
15-AF	1.18	1.76	0.59	2.59	7.86	5.16	2.70	53.8	26.8
Adult male	1.57	1.57	0.66	3.00	8.72	5.72	3.00	70.4	35.6

Heart and contents. The outer surface of the heart is represented by four quarter-ellipsoids. Within this space, the heart is divided into regions representing the muscular walls and the four chambers. The equations are given below in (x_1, y_1, z_1)-coordinates, which are related to the (x, y, z)-coordinate system by the following rotation-translation equations:

$$\begin{bmatrix} x-1 \\ y-1 \\ z_1 \end{bmatrix} = \begin{bmatrix} \alpha_1 & \beta_1 & \gamma_1 \\ \alpha_2 & \beta_2 & \gamma_2 \\ \alpha_3 & \beta_3 & \gamma_3 \end{bmatrix} \begin{bmatrix} x - x_0 \\ y - y_0 \\ z - z_0 \end{bmatrix}$$

In the equations below, the variable names VX, AVY, LAVZ, RAVZ, AX, TLVW, TRVW, and TAW are acronyms in which the letters L and R refer to left and right, A and V to atrium and ventricle, T to thickness, W to wall, and X, Y, and Z to dimensions in the x_1, y_1, and z_1 directions. Thus, AVY is a dimension common to the atria and ventricles in the y_1 direction.

The left ventricle (wall + contents) is represented by half an ellipsoid. The wall is defined by the inequalities

$$\left(\frac{x_1}{VX}\right)^2 + \left(\frac{y_1}{AVY}\right)^2 + \left(\frac{z_1}{LAVZ}\right)^2 \le 1,$$

$$\left(\frac{x_1}{VX - TLVW}\right)^2 + \left(\frac{y_1}{AVY - TLVW}\right)^2 + \left(\frac{z_1}{LAVZ - TLVW}\right)^2 \ge 1,$$

$$and \quad x_1 \ge 0.$$

The contents of the left ventricle are defined by the volume within the inner of the two half-ellipsoids given above, i.e.,

$$\left(\frac{x_1}{VX - TLVW}\right)^2 + \left(\frac{y_1}{AVY - TLVW}\right)^2 + \left(\frac{z_1}{LAVZ - TLVW}\right)^2 < 1,$$

$$and \quad x_1 \ge 0.$$

The right ventricle (wall + contents) is represented by a quarter-ellipsoid that wraps around half of the left ventricle. The wall is defined by the inequalities

$$\left(\frac{x_1}{VX}\right)^2 + \left(\frac{y_1}{AVY}\right)^2 + \left(\frac{z_1}{RAVZ}\right)^2 \le 1,$$

$$\left(\frac{X_1}{VX - TRVW}\right)^2 + \left(\frac{y_1}{AVY - TRVW}\right)^2 + \left(\frac{z_1}{RAVZ - TRVW}\right)^2 \ge 1,$$

$$x_1 \ge 0, \quad and \quad z_1 < 0.$$

The volume common to the left and right ventricle walls is considered part of the left ventricle wall and is excluded here.

The contents of the right ventricle are defined by the inequalities

$$\left(\frac{x_1}{VX - TRVW}\right)^2 + \left(\frac{y_1}{AVY - TRVW}\right)^2 + \left(\frac{z_1}{RAVZ - TRVW}\right)^2 < 1,$$

$$x_1 \ge 0 \ and \quad z_1 < 0.$$

The portion of the left ventricle within this space is excluded, i.e., the inequality must also hold.

$$\left(\frac{x_1}{VX}\right)^2 + \left(\frac{y_1}{AVY}\right)^2 + \left(\frac{z_1}{LAVZ}\right)^2 > 1$$

The left atrium (wall + contents) is represented by two adjacent quarter-ellipsoids. The left atrial wall is defined as follows:

(part 1)

$$\left(\frac{x_1}{AX}\right)^2 + \left(\frac{y_1}{AVY}\right)^2 + \left(\frac{z_1}{LAVZ}\right)^2 \leq 1,$$

$$\left(\frac{x_1}{AX - TAW}\right)^2 + \left(\frac{y_1}{AVY - TAW}\right)^2 + \left(\frac{z_1}{LAVZ - TAW}\right)]^2 \geq 1,$$

$$x_1 < 0 \text{ and } z_1 \geq 0;$$

and (part 2)

$$\left(\frac{x_1}{AX}\right)^2 + \left(\frac{y_1}{AVY}\right)^2 + \left(\frac{z_1}{LAVZ - TLVW + TAW}\right)^2 \leq 1,$$

$$\left(\frac{x_1}{AX - TAW}\right)^2 + \left(\frac{y_1}{AVY - TAW}\right)^2 + \left(\frac{z_1}{LAVZ - TLVW}\right)^2 \geq 1,$$

$$x_1 < 0 \text{ and } z_1 < 0.$$

The contents of the left atrium are represented by the volume within these walls, i.e., by

(part 1)

$$\left(\frac{x_1}{AX - TAW}\right)^2 + \left(\frac{y_1}{AVY - TAW}\right)^2 + \left(\frac{z_1}{LAVZ - TAW}\right)^2 < 1,$$

$$x_1 < 0 \text{ and } z_1 \geq 0;$$

and (part 2)

$$\left(\frac{x_1}{AX - TAW}\right)^2 + \left(\frac{y_1}{AVY - TAW}\right)^2 + \left(\frac{z_1}{LAVZ - TLVW}\right)^2 < 1,$$

$$x_1 < 0 \text{ and } z_1 < 0.$$

The right atrium (wall + contents) is represented by a quarter-ellipsoid that wraps around part of the left atrium. The wall is defined by the inequalities

$$\left(\frac{x_1}{AX}\right)^2 + \left(\frac{y_1}{AVY}\right)^2 + \left(\frac{z_1}{RAVZ}\right)^2 \le 1,$$

$$\left(\frac{x_1}{AX - TAW}\right)^2 + \left(\frac{y_1}{AVY - TAW}\right)^2 + \left(\frac{z_1}{RAVZ - TAW}\right)^2 \ge 1,$$

$$x_1 < 0 \text{ and } z_1 < 0.$$

The volume common to the left and right atrial walls is considered part of the left atrial wall and is excluded here.

The contents of the right atrium are defined by the inequalities

$$\left(\frac{x_1}{AX - TAW}\right)^2 + \left(\frac{y_1}{AVY - TAW}\right)^2 + \left(\frac{z_1}{RAVZ - TAW}\right)^2 < 1,$$

$$x_1 < 0 \text{ and } z_1 < 0.$$

The portion of the left atrium within this space is excluded, i.e., the inequality

$$\left(\frac{x_1}{AX}\right)^2 + \left(\frac{y_1}{AVY}\right)^2 + \left(\frac{z_1}{LAVZ - TLVW + TAW}\right)^2 > 1$$

must also hold.

The age-dependent values of all the heart parameters are given in the tables below. The volumes are given in cubic centimeters.

Phantom	α_1	β_1	γ_1	α_2	β_2	γ_2	α_3	β_3	γ_3
Newborn	0.5942	-0.6421	-0.4845	-0.3291	0.3556	-0.8748	0.7340	0.6792	0
Age 1	0.6009	-0.6216	-0.5025	-0.3493	0.3613	-0.8646	0.7190	0.6950	0
Age 5	0.6237	-0.5721	-0.5327	-0.3926	0.3601	-0.8463	0.6760	0.7369	0
Age 10	0.6345	-0.5370	-0.5559	-0.4243	0.3591	-0.8312	0.6460	0.7633	0
15-AF	0.6453	-0.5134	-0.5658	-0.4428	0.3523	-0.8245	0.6226	0.7825	0
Adult male	0.6751	-0.4727	-0.5664	-0.4640	0.3249	-0.8241	0.5736	0.8191	0

Phantom	VX	AVY	LAVZ	RAVZ	AX	TLVW	TRVW	TAX	x_0	y_0	z_0
Newborn	3.71	2.16	1.34	3.02	2.33	0.56	0.26	0.13	0.42	-1.08	16.05
Age 1	4.67	2.72	1.68	3.80	2.93	1.71	0.33	0.16	0.54	-1.67	22.43
Age 5	5.72	3.33	2.06	4.66	3.59	0.86	0.40	0.20	0.77	-1.70	29.60
Age 10	6.73	3.92	2.43	5.48	4.23	1.02	0.47	0.23	0.80	-1.70	36.60
15-AF	7.86	4.57	2.83	6.40	4.94	1.19	0.55	0.27	0.86	-2.10	45.10
Adult male	8.60	5.00	3.10	7.00	5.40	1.30	0.60	0.30	1.00	-1.80	50.00

Phantom	Volume (cm^3)			
	Left ventricle		Right ventricle	
	Wall	Contents	Wall	Contents
Newborn	14.3	8.23	5.42	8.68
Age 1	28.5	16.2	10.9	17.3
Age 5	52.0	30.2	19.8	32.0
Age 10	85.4	48.9	32.3	52.0
15-AF	135	77.4	51.4	82.9
Adult male	177	102	67.2	108

Phantom	Volume (cm^3)			
	Left atrium		Right atrium	
	Wall	Contents	Wall	Contents
Newborn	2.55	9.31	2.21	8.91
Age 1	4.96	18.5	4.32	18.0
Age 5	9.31	34.0	8.09	32.7
Age 10	14.9	55.8	12.9	53.8
15-AF	23.7	88.3	20.7	85.5
Adult male	31.6	115	27.4	111

Phantom	Volume (cm^3)	
	Walls	Contents
Newborn	24.4	35.1
Age 1	48.7	69.9
Age 5	89.3	129
Age 10	145	210
15-AF	231	334
Adult male	303	437

Kidneys. Each kidney is an ellipsoid cut by a plane, given by the following:

$$\left(\frac{x - x_0}{a}\right)^2 + \left(\frac{y - y_0}{b}\right)^2 + \left(\frac{z - z_0}{c}\right)^2 \le 1$$

$$and \quad |x| \ge x_1.$$

In the following table, x_0 is taken as positive for the left kidney, and negative for the right - the volume is that for both kidneys.

Phantom	a	b	c	x_0	y_0	z_0	x_1	Volume (cm^3)
Newborn	1.79	0.93	1.70	±1.91	2.94	10.03	0.71	22.0
Age 1	2.61	1.25	2.41	±2.64	3.90	14.25	0.95	60.5
Age 5	3.20	1.40	3.20	±3.44	4.50	18.94	1.31	111
Age 10	3.66	1.47	3.99	±4.17	5.04	23.59	1.74	166
15-AF	4.05	1.53	4.96	±5.18	5.88	29.30	2.48	238
Adult male	4.50	1.50	5.50	±6.00	6.00	32.50	3.00	288

Liver. The liver is defined by an elliptical cylinder cut by a plane as follows:

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 \le 1,$$

$$\frac{x}{x_m} + \frac{y}{y_m} - \frac{z}{z_m} \le -1, \; and \; z_1 \le z \le z_2.$$

The liver in the age-15-male/adult-female phantom has been changed slightly from that given for the age 15 phantom in Cristy (1980) to match the data for a reference adult female (ICRP 1975).

Phantom	a	b	x_m	y_m	z_m	z_1	z_2
Newborn	5.19	4.25	8.45	10.90	13.27	8.33	13.27
Age 1	7.20	5.47	12.83	16.55	18.86	11.84	18.86
Age 5	9.39	6.30	16.27	20.34	25.06	15.74	25.06
Age 10	11.43	6.83	21.98	29.67	31.21	19.59	31.21
15-AF	14.19	7.84	31.51	44.75	38.76	24.34	38.76
Adult male	16.50	8.00	35.00	45.00	43.00	27.00	43.00

Phantom	Volume (cm^3)
Newborn	117
Age 1	281
Age 5	562
Age 10	853
15-AF	1350
Adult	1830

Lungs. Each lung is represented by half an ellipsoid with a section removed. Note that the section removed from the left lung is larger than that removed from the right lung because of the position of the heart. The right lung is defined as follows:

$$\left(\frac{x + x_0}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z - z_0}{c}\right)^2 \le 1 \text{ and } z \ge z_0$$

if $z_{1R} \le z \le z_{2R}$ *and* $y < y_{1R}$ *, then* $x \le x_{1R}$ *must also hold.*

The statements for the left lung are similar, but replace $(x + x_0)$ with $(x - x_0)$, and z_{1R}, z_{2R}, and y_{1R} with z_0, z_{2L}, and y_{1L}, respectively; and replace the inequality $(x \le x_{1R})$ with $(x \ge x_{1L})$. The letters *R* and *L* refer to right and left.

Phantom	*a*	*b*	*c*	x_0	z_0
Newborn	1.89	3.68	7.41	2.70	13.42
Age 1	2.68	4.88	10.53	3.74	19.08
Age 5	3.47	5.63	13.99	4.87	25.35
Age 10	3.82	6.30	17.42	5.91	31.57
15-AF	4.09	6.98	20.55	7.33	39.21
Adult male	5.00	7.50	24.00	8.50	43.50

Phantom	x_{1R}	y_{1R}	z_{1R}	z_{2R}	x_{1L}	y_{1L}	z_{2L}
Newborn	-2.30	0.75	14.15	17.85	+3.00	0.30	17.90
Age 1	-2.90	0.70	20.10	24.60	+3.90	0.40	24.80
Age 5	-3.50	1.00	26.90	32.30	+5.00	0.50	32.60
Age 10	-4.10	1.30	33.40	39.60	+5.90	0.75	40.00
15-AF	-5.00	1.20	41.60	48.50	+7.00	0.70	49.00
Adult male	-5.40	1.50	46.00	54.00	+8.00	1.00	55.00

Phantom	Volume (cm^3)		
	Left lung	Right lung	Both lungs
Newborn	79.1	91.9	171
Age 1	225	259	484
Age 5	454	526	980
Age 10	709	821	1530
15-AF	1020	1180	2200
Adult male	1560	1810	3380

Ovaries. Each ovary is an ellipsoid and is given by

$$\left(\frac{x - x_0}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z - z_0}{c}\right)^2 \le 1.$$

The values of x_0 in the table below are taken as positive for the left ovary, and negative for the right ovary.

Phantom	a	b	c	x_0	z_0	Volume (cm^3)
Newborn	0.30	0.22	0.57	±1.91	4.63	0.315
Age 1	0.38	0.28	0.77	±2.64	6.58	0.686
Age 5	0.53	0.35	1.07	±3.44	8.74	1.66
Age 10	0.66	0.40	1.36	±4.17	10.89	3.01
15-AF	1.17	0.58	1.80	±5.18	13.52	10.2
Adult male	1.00	0.50	2.00	±6.00	15.00	8.38

Pancreas. The pancreas is half an ellipsoid with a section removed. It is defined by

$$\left(\frac{x - x_0}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z - z_0}{c}\right)^2 \le 1,$$

$$x \ge x_0 \text{ and } z \ge z_0 \text{ if } x > x_1.$$

Phantom	a	b	c	x_0	z_0	x_1	Volume (cm^3)
Newborn	4.32	0.50	0.87	-0.09	11.42	0.99	2.69
Age 1	6.85	0.71	1.41	-0.43	16.23	1.32	9.87
Age 5	9.16	0.90	1.92	-0.57	21.57	1.72	22.7
Age 10	10.09	0.92	2.17	-0.38	26.85	2.15	28.9
15-AF	13.32	1.14	2.87	-0.72	33.35	2.61	62.4
Adult male	16.00	1.20	3.30	-1.00	37.00	3.00	90.7

Skin. Skin is represented as a layer of thickness S extending over the exterior of the phantom, including the exposed top of the trunk and the bottom of the legs, but excluding the exposed bottom of the trunk, top of the legs, and bottom of the head. The part of the legs covered by the male genitalia region has skin, but the part of the trunk covered by the female breasts does not. The skin layer corresponds to the dermis as well as the epidermis. Greater thicknesses in places such as the back have been ignored.

		Volume of skin (cm^3)				
Phantom	S	Head	Trunk	Legs	Male Genitalia	Total
Newborn	0.07	30.2	54.6	28.3	0.741	119
Age 1	0.08	66.4	128	75.0	1.48	271
Age 5	0.09	101	238	195	2.64	536
Age 10	0.10	127	385	363	4.05	879
15-AF	0.17	251	1000	879	13.5	2130
Adult male	0.20	318	1440	1210	23.4	2890

Spleen. The spleen is represented by the ellipsoid

$$\left(\frac{x - x_0}{a}\right)^2 + \left(\frac{y - y_0}{b}\right)^2 + \left(\frac{z - z_0}{c}\right)^2 \le 1.$$

Phantom	a	b	c	x_0	y_0	z_0	Volume (cm^3)
Newborn	1.13	1.00	1.85	3.54	1.42	11.42	8.76
Age 1	1.65	1.35	2.63	4.94	1.85	16.23	24.5
Age 5	2.09	1.52	3.49	6.40	2.25	21.57	46.4
Age 10	2.43	1.68	4.35	7.65	2.52	26.85	74.4
15-AF	2.90	1.88	5.19	9.49	2.94	33.35	119
Adult male	3.50	2.00	6.00	11.00	3.00	37.00	176

Testes. The testes are represented by the ellipsoids

$$\left(\frac{x \pm a}{a}\right)^2 + \left(\frac{y - y_0}{b}\right)^2 + \left(\frac{z + c}{c}\right)^2 \le 1,$$

where the ± sign is taken as positive for the right testis and negative for the left testis.

Phantom	a	b	c	y_0	Volume (cm^3)
Newborn	0.36	0.42	0.64	-2.58	0.811
Age 1	0.41	0.47	0.72	-3.73	1.16
Age 5	0.45	0.52	0.80	-4.98	1.57
Age 10	0.47	0.55	0.84	-6.15	1.82
15-AF	0.96	1.10	1.69	-7.10	15.0
Adult male	1.30	1.50	2.30	-8.00	37.6

Thymus. The thymus is represented by an ellipsoid, given by

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y - y_0}{b}\right)^2 + \left(\frac{z - z_0}{c}\right)^2 \le 1.$$

Phantom	a	b	c	y_0	z_0	Volume (cm^3)
Newborn	1.76	0.70	2.10	-3.60	19.30	10.8
Age 1	1.75	1.00	3.00	-4.75	27.00	22.0
Age 5	1.85	1.05	3.50	-5.48	35.00	28.5
Age 10	1.85	1.00	3.90	-6.13	43.00	30.2
15-AF	1.75	0.93	4.00	-7.15	52.00	27.3
Adult male	1.50	0.80	4.00	-7.30	57.00	20.1

Thyroid. The lobes of the thyroid lie between two concentric cylinders and are formed by a cutting surface. The statements defining this organ are

$$x^2 + (y - y_0)^2 \le R^2,$$

$$x^2 + (y - y_0)^2 \ge r^2,$$

$$y \le y_0,\ C_T \le z \le C_T + c,\ \textit{and}\ [(y - y_0) - |x|]^2 \ge 2[x^2 + (y - y_0)^2]\tau^2,$$

in which

$$\tau = \left(\frac{\sqrt{2}-2}{2}\right)\left(\frac{z-C_T}{0.25c}\right)+1 \quad \textit{for } 0 \le z - C_T \le 0.25c$$

$$\textit{and} \quad \tau = \left(\frac{2-\sqrt{2}}{2}\right)\left(\frac{z-C_T}{0.75c}\right)+\frac{2\sqrt{2}-1}{3} \quad \textit{for} \quad 0.25c \le z - C_T \le c.$$

Phantom	R	r	c	y_0	Volume (cm^3)
Newborn	0.87	0.40	2.00	-2.14	1.24
Age 1	0.97	0.44	2.21	-2.87	1.71
Age 5	1.21	0.55	2.76	-3.31	3.32
Age 10	1.60	0.73	3.63	-3.56	7.62
15-AF	1.85	0.83	4.20	-3.91	11.9
Adult male	2.20	1.00	5.00	-4.00	19.9

Urinary bladder and contents. The bladder wall is represented by the volume between two concentric ellipsoids. The contents are represented by the volume within the inner ellipsoid. The wall is defined by

$$\left(\frac{x}{a}\right)^2+\left(\frac{y-y_0}{b}\right)^2+\left(\frac{z-z_0}{c}\right)^2 \le 1$$

$$\textit{and} \left(\frac{x}{a-d}\right)^2+\left(\frac{y-y_0}{b-d}\right)^2+\left(\frac{z-z_0}{c-d}\right)^2 < 1.$$

The contents are defined by

$$\left(\frac{x}{a-d}\right)^2+\left(\frac{y-y_0}{b-d}\right)^2+\left(\frac{z-z_0}{c-d}\right)^2 < 1.$$

							Volume (cm^3)	
Phantom	a	b	c	d	y_0	z_0	Wall	Contents
Newborn	1.69	1.82	1.14	0.10	-2.21	2.47	2.77	11.9
Age 1	2.35	2.42	1.64	0.14	-2.93	3.51	7.41	31.7
Age 5	3.04	2.77	2.16	0.17	-3.38	4.66	14.0	62.2
Age 10	3.61	3.04	2.63	0.20	-3.78	5.81	22.3	98.6
15-AF	4.27	3.38	3.11	0.23	-4.41	7.21	34.5	154
Adult male	4.958	3.458	3.458	0.252	-4.50	8.00	45.7	203

Uterus. The uterus is an ellipsoid cut by a plane and is given by

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y - y_0}{b}\right)^2 + \left(\frac{z - z_0}{c}\right)^2 \leq 1$$

$$and \quad y \geq y_1 .$$

Phantom	a	b	c	y_0	z_0	y_1	Volume (cm^3)
Newborn	0.83	2.57	0.49	-0.98	4.32	-2.27	3.70
Age 1	0.61	1.80	0.36	-1.30	6.14	-2.20	1.40
Age 5	0.78	2.00	0.47	-1.50	8.16	-2.51	2.60
Age 10	0.91	2.17	0.57	-1.68	10.16	-2.78	4.00
15-AF	2.47	5.61	1.55	-1.96	12.62	-4.77	76.0
Adult male	2.62	5.22	1.57	-2.00	14.00	-4.62	76.0

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Table 1. Elemental composition of the tissues for all phantoms.

	All phantoms except newborn			Newborn		
	Percent by weight			Percent by weight		
Element	Soft tissue	Skeleton	Lung	Soft tissue	Skeleton	Lung
H	10.454	7.337	10.134	10.625	7.995	10.134
C	22.663	25.475	10.238	14.964	9.708	10.238
N	2.490	3.057	2.866	1.681	2.712	2.866
O	63.525	47.893	75.752	71.830	66.811	75.752
F	0	0.025	0			
Na	0.112	0.326	0.184	0.075	0.314	0.184
Mg	0.013	0.112	0.007	0.019	0.143	0.007
Si	0.030	0.002	0.006			
P	0.134	5.095	0.080	0.179	3.712	0.080
S	0.204	0.173	0.225	0.240	0.314	0.225
Cl	0.133	0.143	0.266	0.079	0.140	0.266
K	0.208	0.153	0.194	0.301	0.148	0.194
Ca	0.024	10.190	0.009	0.003	7.995	0.009
Fe	0.005	0.008	0.037	0.004	0.008	0.037
Zn	0.003	0.005	0.001			
Rb	0.001	0.002	0.001			
Sr	0	0.003	0			
Zr	0.001	0	0			
Pb	0	0.001	0			
Density (g/cm^3)	1.04	1.4	0.296	1.04	1.22	0.296

Table 2. Active marrow in individual bones, parts of bones, or bone groups expressed as the percentage of active marrow in the body

Phantom skeletal region	Percentage at various ages					
	0	1	5	10	15	Adult[a]
Skull (cranium + facial skeleton)[b]	29.50	27.47	17.44	12.72	10.12	8.32
Scapulae	2.70	2.73	2.72	2.89	3.26	2.85
Clavicles	0.80	0.83	0.85	0.89	0.98	0.79
Ribs	9.20	9.61	10.58	13.02	16.27	19.22
Spine (upper portion[c])	2.30	1.88	1.46	1.80	2.25	2.66
Spine (middle portion[c])	9.40	9.27	9.58	11.79	14.75	17.41
Spine (lower portion[c])	1.90	3.37	5.39	6.63	8.29	9.79
Pelvis	11.66	16.47	23.33	28.73	33.60	33.31
Leg bones (upper portion[d])	1.87	2.07	3.41	4.72	4.60	3.35
Leg bones (middle portion[d])	3.73	3.88	6.28	6.14	2.04	0
Leg bones (lower portion[d])	16.24	13.40	11.55	5.51	0	0
Arm bones (upper portion[e])	2.32	2.41	2.36	2.49	3.14	2.29
Arm bones (upper middle portion[e])	2.32	2.25	2.18	1.62	0.70	0
Arm bones (lower portion[e])	6.07	4.36	2.88	1.06	0	0

[a]Age 40 values from Cristy (1981) were used for the "Adult male" phantom.

[b]In column 1, cranium does not include the facial skeleton, but in column 2, cranium includes all the facial skeleton except the mandible.

[c]The upper, middle, and lower portions of the spine are defined in the section on the spine.

[d]The upper portion of the leg bones is defined as the upper 14% of the length of the bones; the lower portion is defined as the lower 57%; and the middle portion is the rest. The unevenness of these numbers results from the assignment of part of the marrow in the upper femora to the pelvis.

[e]The upper portion of the arm bones is defined as the upper 25% of the length of the bones; the lower portion is defined as the lower 50%; and the middle portion is the rest.

Table 3. Inactive marrow in individual bones, parts of bones, or bone groups expressed as the percentage of inactive marrow in the body (derived from Cristy, 1981)

Phantom skeletal region	Percentage at various ages					
	0	1	5	10	15	Adult[a]
Skull (cranium + facial skeleton)b	0	11.33	10.17	7.16	5.63	6.35
Scapulae	0	1.06	1.60	1.64	1.82	2.17
Clavicles	0	0.35	0.53	0.53	0.61	0.75
Ribs	0	3.90	4.32	3.42	3.70	3.86
Spine (upper portion[c])	0	0.78	0.61	0.47	0.51	0.53
Spine (middle portion[c])	0	3.82	3.95	3.10	3.34	3.50
Spine (lower portion[c])	0	1.40	2.21	1.74	1.90	1.97
Pelvis	0	6.79	13.28	11.87	13.28	16.07
Leg bones (upper portion[d])	0	0.84	2.36	3.31	3.83	4.70
Leg bones (middle portion[d])	0	3.72	5.95	10.08	12.53	12.53
Leg bones (lower portion[d])	0	45.35	39.29	40.05	36.66	32.05
Arm bones (upper portion[e])	0	0.97	1.63	1.74	2.62	3.21
Arm bones (middle portion[e])	0	2.21	2.06	2.64	4.29	4.29
Arm bones (lower portion[e])	0	17.45	12.04	12.25	9.28	8.02

[a]Age 40 values from Cristy (1981) were used for the "Adult male" phantom.

[b]In column 1, cranium does not include the facial skeleton, but in column 2, cranium includes all the facial skeleton except the mandible.

[c]The upper, middle, and lower portions of the spine are defined in the section on the spine.

[d]The upper portion of the leg bones is defined as the upper 14% of the length of the bones; the lower portion is defined as the lower 57%; and the middle portion is the rest. The unevenness of these numbers results from the assignment of part of the marrow in the upper femora to the pelvis.

[e]The upper portion of the arm bones is defined as the upper 25% of the length of the bones; the lower portion is defined as the lower 50%; and the middle portion is the rest.